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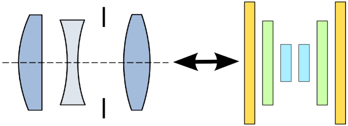
ONERA
THE FRENCH AEROSPACE LAB

End-to-end sensor design: from Co-design to Deep-Codesign



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1. ONERA/DTIS-IVA
2. ONERA/DOTA-ERIO

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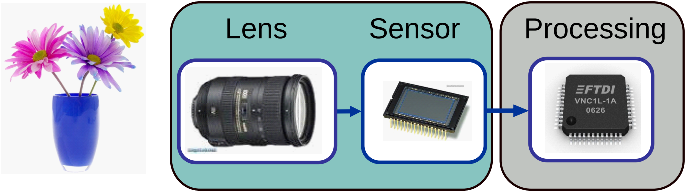
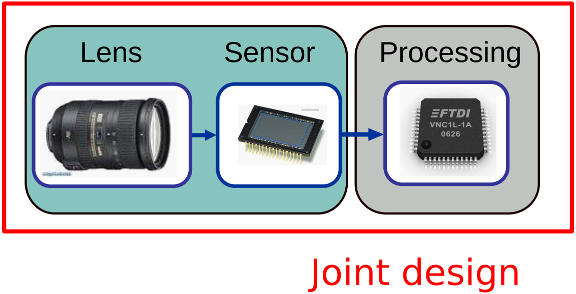
Contents

Contents

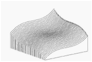
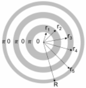


Lens
Sensor
Processing
FTDI
VNC1L-1A
0626


Lens
Sensor
Processing
FTDI
VNC1L-1A
0626
Joint design

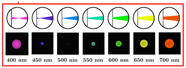
Depth of field extension

Phase mask

[Cathey et Dowski, AO,2002]
[Diaz et al, OL, 2009]

Chromatic aberration



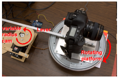
[Cossairt et al, ICCP 2010]

Motion blur removal

Flutter Shutter

[Agrawal et al, CVPR, 2009]

Parabolic translation



[Levin et al, Siggraph 2008]

3D

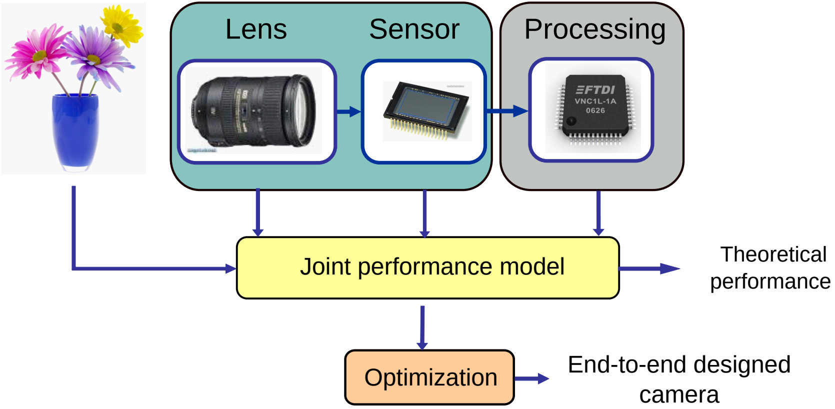
Coded aperture


[Levin et al, Siggraph 2007] [Chakrabarti et al, ECCV, 2012]

Lens with chromatic aberration


[Trouvé et al, AO, 2013] [Trouvé et al, AO, 2018]

$\Longrightarrow$ A "nice image" is no longer required at the lens output !

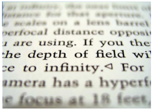


Lens
Sensor
Processing
FTDI
VNC1L-1A
0626
Joint performance model
Theoretical performance
Optimization
End-to-end designed camera

Contents

Contents

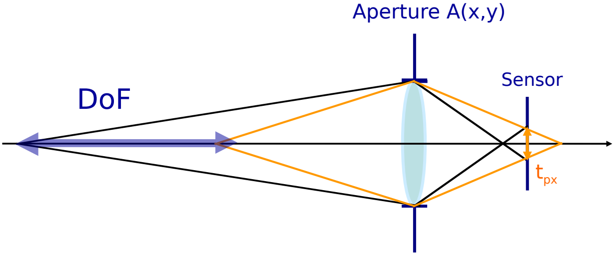
What is depth of field extension ?



Source : wikipedia

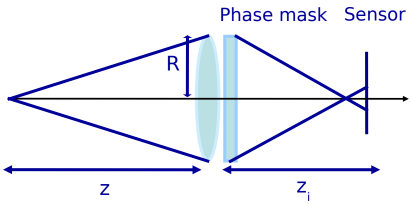
Depth of field (DoF)

Region where objects have a sharp image



$\Longrightarrow$ EDOF: extension of the DoF region

Principle A phase mask to reduce PSF variation vs depth [1]



- Without the phase mask:

$$h_\psi(x_P, y_P) \propto \left| \mathrm{FT}\left(A(x,y)\exp^{i\psi\frac{(x^2+y^2)}{R^2}} \right) \right|^2_{\left(\frac{x_P}{\lambda z_i}, \frac{y_P}{\lambda z_i}\right)} \text{ with } \psi = \frac{\pi R^2}{\lambda}\left(\frac{1}{z} + \frac{1}{z_i} - \frac{1}{f}\right)$$

- With the phase mask :

$$h_\psi(x_P, y_P) \propto \left| \mathrm{FT}\left(A(x,y)\exp^{i\psi\frac{(x^2+y^2)}{R^2}} \exp^{i\phi_{\mathrm{mask}}(x,y)} \right) \right|^2_{\left(\frac{x_P}{\lambda z_i}, \frac{y_P}{\lambda z_i}\right)}$$

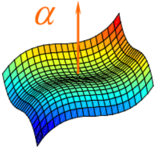
[1] Dowski, E. R. and Cathey, W. T. Appl. Opt., 1995, **34**

Co-design of a phase mask for EDOF

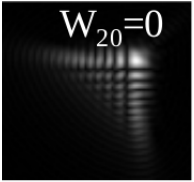
Example of phase mask

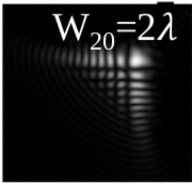
- Cubic phase plate [1, 2]

  $\phi_{mask}(x, y) = \alpha(x^3 + y^3)$



PSF at various depths ($W = \frac{\psi}{2\pi}$):





$\Longrightarrow$ Image restoration is required !

[1] Dowski, E. R. and Cathey, W. T. Appl. Opt., 1995, **34**
[2] F. Diaz et al., Opt. Lett., (2009) **34**

Image formation model : $y = h * x + n$

Scene and noise PSD : PSD_x and PSD_n

Restoration filter for a discrete set of depths [2]
A unique deconvolution filter for all the depths (mean Wiener filter) :

$$\tilde{d}^{\alpha}(\mu,\nu) = \frac{\frac{1}{K}\sum_{k=1}^{K}\tilde{h}^{\alpha}_{\psi_k}(\mu,\nu)\mathrm{PSD}_x(\mu,\nu)}{\frac{1}{K}\sum_{k=1}^{K}|\tilde{h}^{\alpha}_{\psi_k}(\mu,\nu)|^2\mathrm{PSD}_x(\mu,\nu)+\mathrm{PSD}_n(\mu,\nu)}$$

Performance model of image restoration at depth k

$$\mathrm{MSE}(\alpha,\psi_k) = \int_{\mu}|\tilde{d}^{\alpha}\tilde{h}^{\alpha}_{\psi_k}-1|^2\mathrm{PSD}_x(\mu)d\mu + \int_{\mu}\mathrm{PSD}_n|\tilde{d}^{\alpha}|^2(\mu)d\mu$$

Expression in terms of image quality

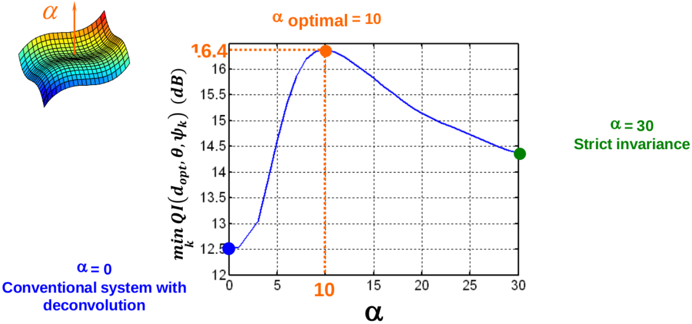
$$\mathrm{IQ}(\alpha,\psi_k) = \frac{E_x}{\mathrm{MSE}(\alpha,\psi_k)} \text{ with } E_x \text{ the energy of the scene}$$

Co-design of a phase mask for EDOF

Optimization criterion

Design criterion for a discrete set of depths $\{k\}$

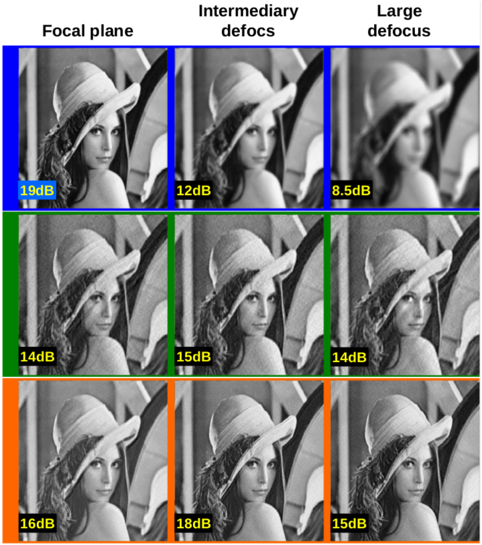
$$\widehat{\alpha} = \arg\max_{\alpha} \arg\min_{k} \mathrm{IQ}(\alpha, \psi_k)$$

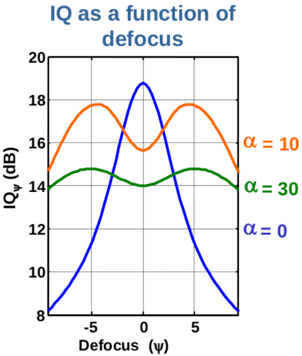


$\Longrightarrow$ Trade off between strictly invariant PSF and image quality

Co-design of a phase mask for EDOF

Deconvolution results

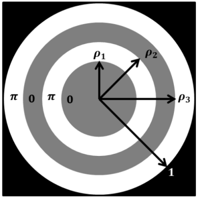


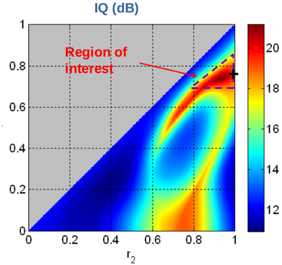


Co-design of phase mask for EDOF

Other phase masks

- Annular phase mask - further works at IOGS



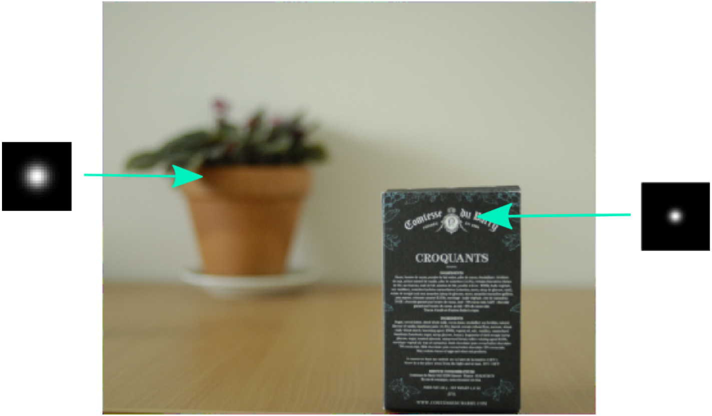


- Optimization of the number of rings [3]
- Application to 3D particle microscopy [4]
- Experimental validation and application to panchromatic illumination [5]

[3] R. Falcon et al., Imaging and Applied Optics 2016
[4] O. Leveque et al., Opt. Express, 2020, **28**
[5] A. Fontbonne et al., SPIE, 2021, **60**.

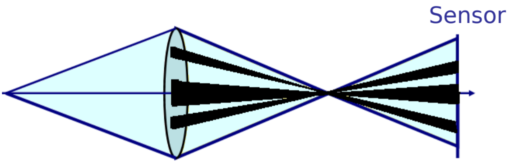
Contents

Monocular 3D camera using Depth from Defocus

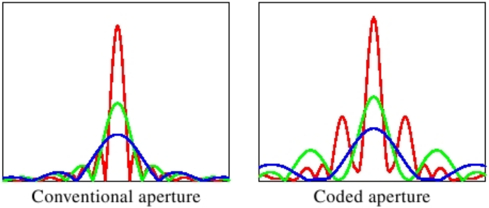


- Depth can be estimated from local defocus blur estimation
- Compact 3D camera - but tricky processing
- How to modify the optics to improve defocus blur estimation ?

Coded aperture (2007)[6] for a better defocus blur discriminiation



MTF= $|\tilde{h}|$ at various depths



[6] A. Levin et al. ACM SIGGRAPH, 2007

Image formation model $y = h * x + n$

Scene gaussian model

$$p(\mathbf{x}) \propto \exp -\frac{\alpha}{2}\left(|F_v\mathbf{x}|^2 + |F_h\mathbf{x}|^2\right)$$

F_h and F_v convolution matrices relative to image gradients

Noise gaussian model : White gaussian noise of variance σ_n^2

Image likelihood in Fourier domain for depth k

$$p_k(\tilde{y}) \propto \exp -\frac{1}{2}\sum_{\mu,\nu}\frac{|\tilde{y}(\mu,\nu)|^2}{\sigma_k(\mu,\nu)}$$

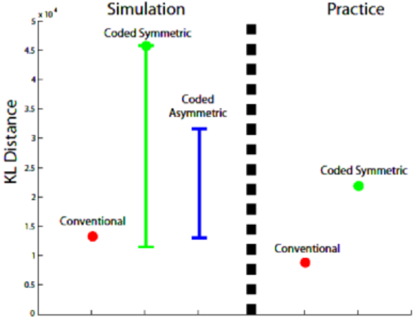
$$\sigma_k(\mu,\nu) = |\tilde{h}_k(\mu,\nu)|^2\left(\alpha|\tilde{F}_1(\mu,\nu)|^2 + \alpha|\tilde{F}_2(\mu,\nu)|^2\right)^{-1} + \sigma_n^2$$

Coded aperture

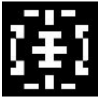
Optimization criterion

Principle : Maximisation of Kullback-Leibler divergence between depths

$$D_{KL}(p_{k_1}||p_{k_2}) = \sum_{\mu,\nu} \left(\frac{\sigma_{k_1}(\mu,\nu)}{\sigma_{k_2}(\mu,\nu)} - \log \frac{\sigma_{k_1}(\mu,\nu)}{\sigma_{k_2}(\mu,\nu)} \right)$$

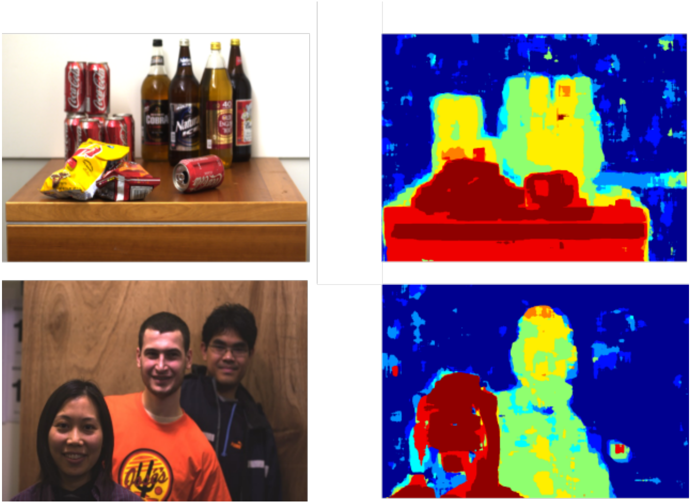


Optimal aperture



Coded aperture

Depth estimation results

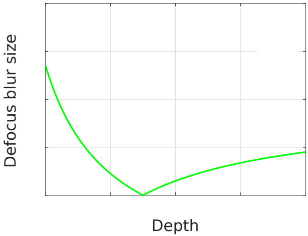


Contents

Monocular 3D using depth from defocus

Issues

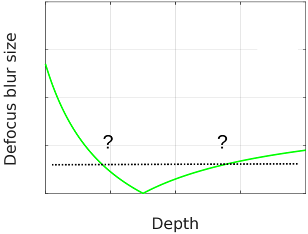
Defocus blur variation for a conventional lens



Monocular 3D using depth from defocus

Issues

Defocus blur variation for a conventional lens

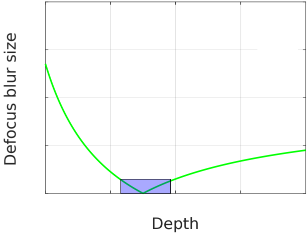


Same blur level for two depths

Monocular 3D using depth from defocus

Issues

Defocus blur variation for a conventional lens



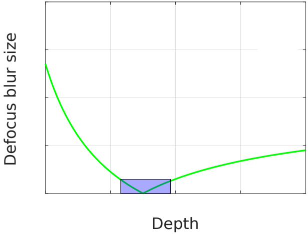
Same blur level for two depths

No blur variation within the camera depth of field

Monocular 3D using depth from defocus

Issues

Defocus blur variation for a conventional lens

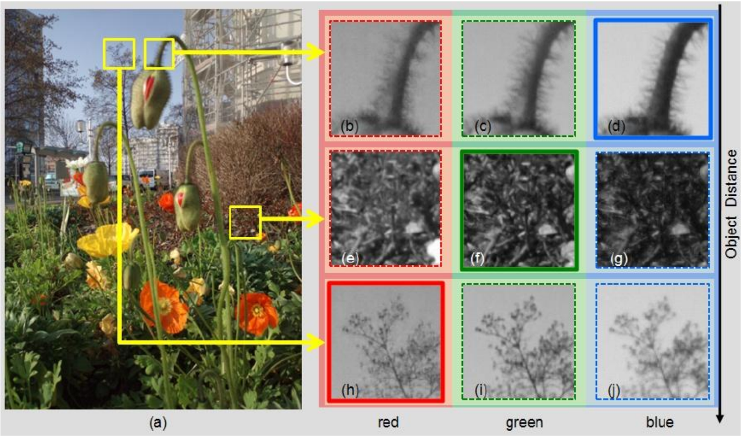


Same blur level for two depths
No blur variation within the camera depth of field

But chromatic aberration can overcome these issues !

Lens with chromatic aberration
Principle

Variation of the focal length with the wavelength

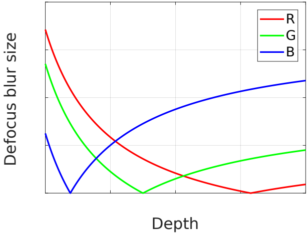


Extracted from Guichard et al., SPIE (2009) [7]

Lens with chromatic aberration

Benefit for depth from defocus

Defocus blur variation for a lens with chromatic aberration

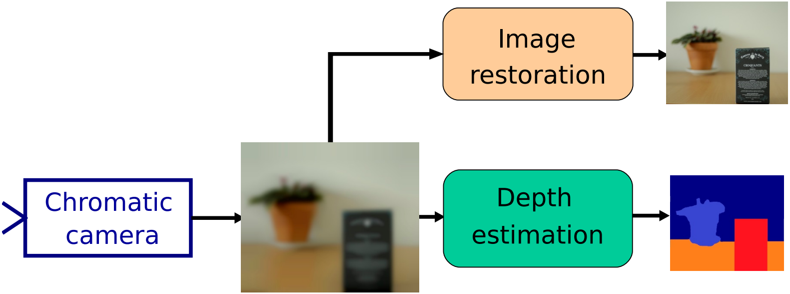


- No dead zone
- No depth ambiguity
- Reduces image quality

Co-design of a monocular RGB-D camera

Framework

Work developed at ONERA [8, 9]



Two outputs lead to two performance models for co-design

[8] P. Trouvé et al., IEEE CVPR Workshops on Comp. Cameras and Displays, (2013)
[9] P. Trouvé-Peloux et al., J. Opt. Soc. Am. A 38(10) (2021)

Image formation model $\mathbf{y} = H(z)\mathbf{x} + \mathbf{n}$

Gaussian scene model

$$p(\mathbf{x}) \propto \exp - \frac{||D\mathbf{x}||^2}{2\sigma_x^2}$$

D concatenation of convolution matrices relative to image gradients (h and v)

Noise gaussian model : White gaussian noise of variance σ_n^2

Image likelihood

$$p(\mathbf{y}; z) = \left|\frac{Q(z)}{2\pi}\right|_+^{1/2} \exp - \frac{1}{2}\mathbf{y}^t Q(z)\mathbf{y}$$

With $Q(z) = \frac{1}{\sigma_b^2}(I - H(z)(H^t(z)H(z) + \alpha D^t D)^{-1} H^t(z))$ and $\alpha = \sigma_b^2/\sigma_x^2$

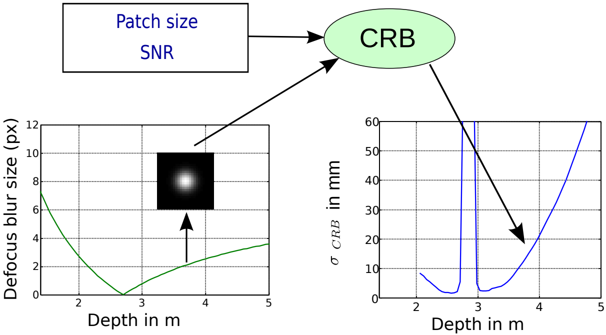
Performance model Cramér Rao Lower Bound

$$\sigma_{CRB}^2(z) = \left(\frac{1}{2}\mathrm{tr}\left(Q^+(z)\frac{\partial Q}{\partial z}Q^+(z)\frac{\partial Q}{\partial z}\right)\right)^{-1}$$

$|A|_+$ product of non zeros eigenvalues of matrix A and A^+ pseudo-inverse of A

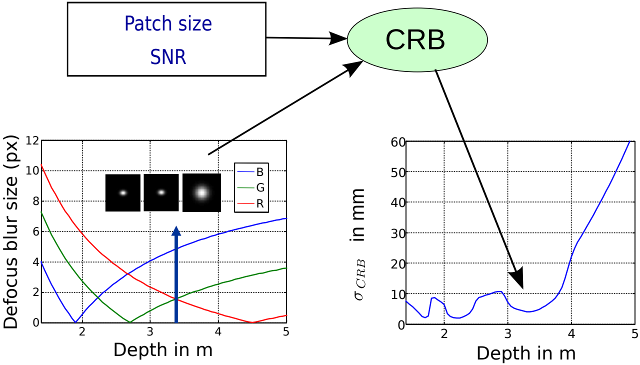
Co-design of a monocular RGB-D camera

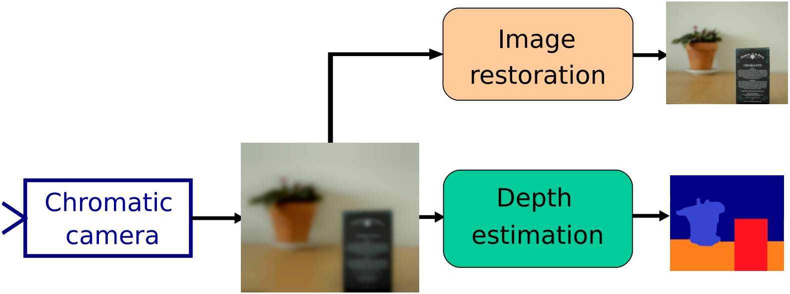
Performance model for DFD for a conventional lens



- Theoretical depth estimation accuracy for a given depth
- With respect to SNR value
- With respect to a given lens

Principle: use a Gaussian scene model in the luminance/chrominance decomposition





Two outputs lead to two performance model for co-design

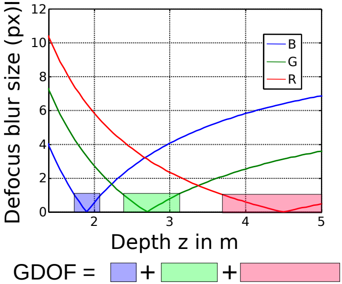
[8] P. Trouvé et al., IEEE CVPR Workshops, (2013)
[9] P. Trouvé-Peloux et al., J. Opt. Soc. Am. A 38(10) (2021)

Principle

- With chromatic aberration there is a "sharpest" channel
- High frequencies transfer to the blurred channels [7]
- Use of the depth map to guide the transfer [9]

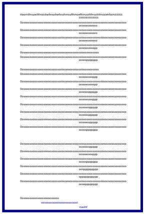
$$y_c = \alpha(z)\mathrm{HF}_R + \beta(z)\mathrm{HF}_G + \gamma(z)\mathrm{HF}_B$$

- Definition of the Generalized Depth of Field (GDOF) :



Application to small UAV navigation

- Depth range 1 to 5 m
- Field of view 25°
- Spatial depth map resolution 2cm at 3 m
- Accuracy of few cm



Settings

?

Several parameters are fixed from the settings

Settings

↓

Fixed parameters

Sensor Stingray (pixel 3.45 µm)

Focal length 24 mm

F-number 3

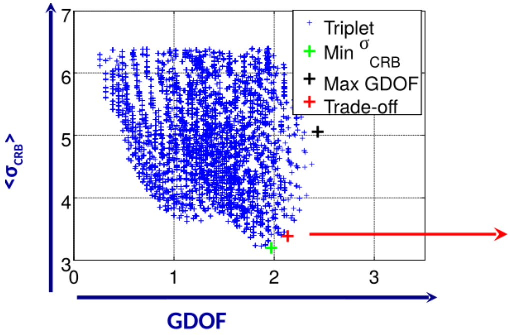
Patch size 45x45 pixels

Chromatic aberration ?

Focus ?

Principle

- Simulation of optical systems with geometrical optics (Gaussian PSF)
- Performance comparison of various systems



Trade-off

Chromatic axial aberration≈130 µm

RGB in-focus planes

[2.2 m 3.2 m 4.2 m]

Settings

Fixed parameters

Sensor Stingray (pixel 3.45 µm)

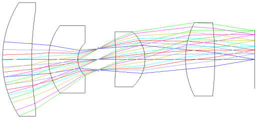
Focal length 24 mm

F-number 3

Patch size 45x45 pixels

Chromatic aberration 130 µm

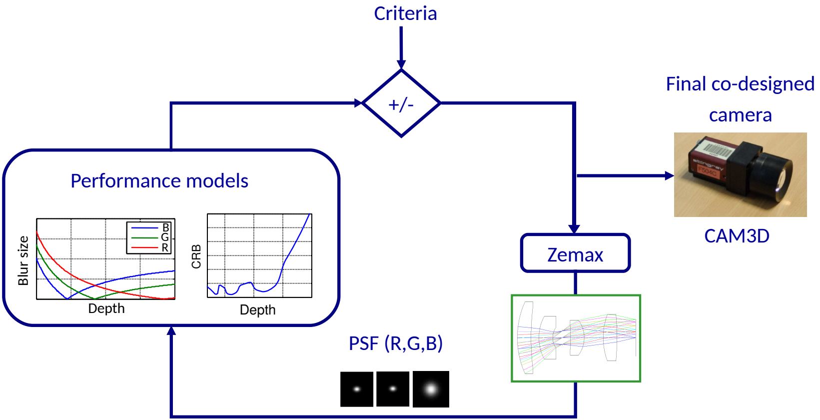
Focus BGR [2.2 - 3.2 - 4.2] m



Architecture design on Zemax

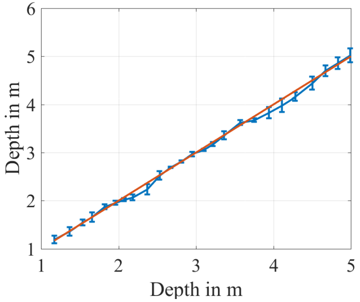
Co-design of a monocular RGB-D camera
Finer optimization

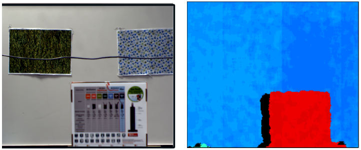
- Design of a first architecture using Zemax
- Fine tuning using performance models and PSF from Zemax



Co-design of a monocular RGB-D camera
Experimental results

Depth estimation results





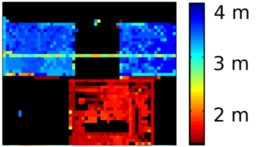
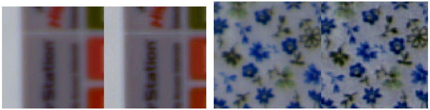


Image restoration results



Contents

Conclusion

States of the art of co-design using performance models

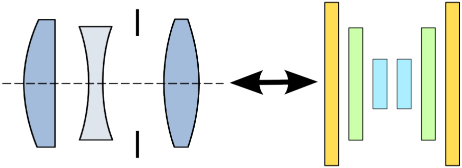
Common approach

- Choice of an (un)conventional optical model
  (Fourier optics, geometrical optics, optical design software...)
- Choice of a processing model
- Definition of a performance model
  (CRB, RMSE, KL divergence...)
- Joint optimization of the parameters
- Experimental validation

Performance model enables the analysis of the interaction between lens properties and processing

Conclusion

States of the art of **("old school")** co-design using performance models



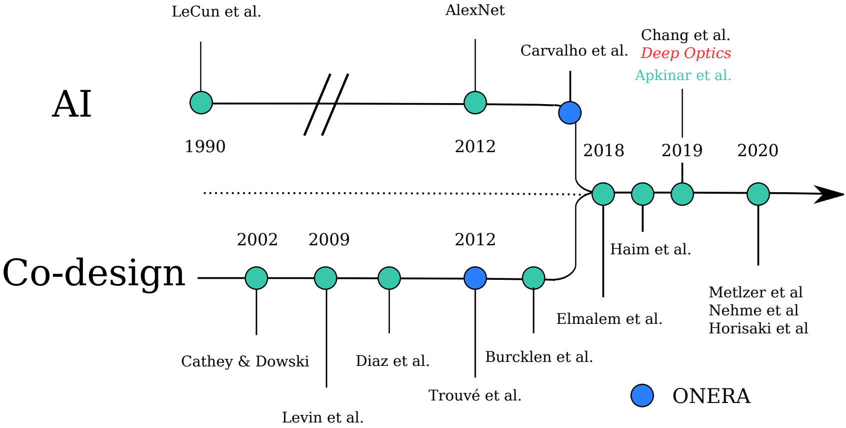
How to make co-design with a neural network ?

Contents

Contents

Deep co-design

State of the art - not exhaustive !



Several fields of application :

- Depth of field extension [10, 11, 12]
- Lateral field of view extension [13]
- 3D [14, 15, 16, 17, 18]
- Object classification [19, 15]
- HDR [20]
- Lensless imaging [21]

Unconventional lens :

- Phase mask [10, 11, 12, 14, 16]
- Coded aperture [21]
- *Freeform* lens [15]

⇒ How to conduct joint lens and network optimization ?

Contents

Deep codesign

Generic framework

Image formation model $y = h(\phi) * x + \eta$

$\Rightarrow$ A lens can be modeled using a convolutional layer of a neural network

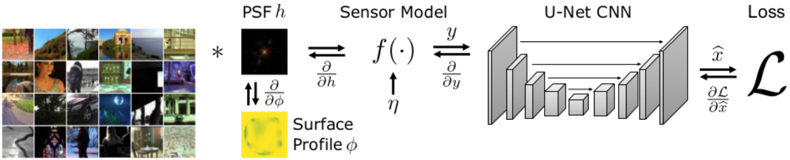


Figure extracted from Metzler et al, CVPR 2020 [20]

> End-to-end optimization can be conducted as long as the gradient of h (i.e. the PSF) can be defined with respect to ϕ

A more detailed example : the presentation of E. Sahim yesterday !

Contents

Contents

State of the art papers

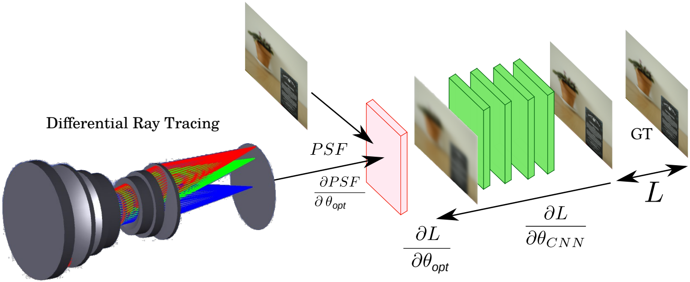
- Optical model based on Fourier Optics (thin lens and paraxial rays approx.)
- Optimization of a single optical element (phase mask, single lens...)

Our approach

Optical model based on differential ray tracing (DRT) [22]

- No thin lens nor paraxial approximation
- Optimization possible of the full set of lens parameters θ_{opt}
- Valid for any field of view and point source depth (α,z)

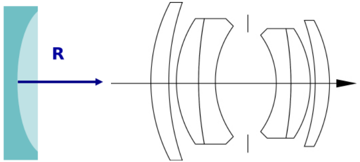
[22] Halé et al., Optics. Express (to appear), 2021

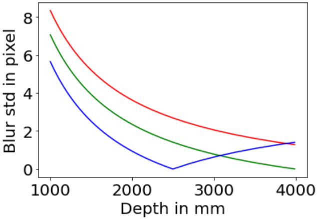


Differential Ray Tracing
PSF
$\frac{\partial PSF}{\partial \theta_{opt}}$
GT
L
$\frac{\partial L}{\partial \theta_{CNN}}$
$\frac{\partial L}{\partial \theta_{opt}}$

Contents

Add-on to add **chromatic aberration** to an existing conventional lens [23]



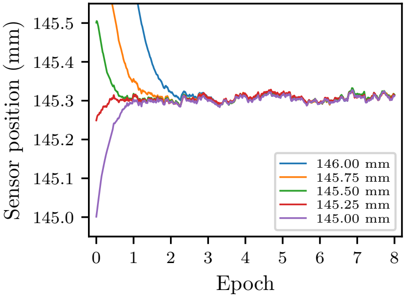


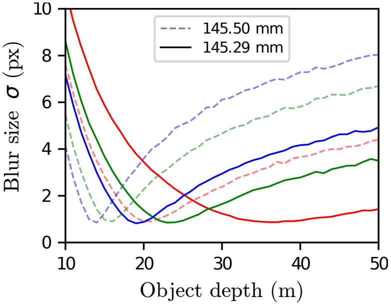
Settings

- Add-on in front of a Double Gauss lens
- A state of the art restoration neural network [24]
- Texture database [25] (patch 64x64)
- **Single** degree of freedom : the sensor position (RGB focus)

[23] Trouvé-Peloux et al. Applied Optics, 57, 2018.
[24] Mao et al. Advances in neural information processing systems, 2016.
[25] Cimpoi et al., IEEE CVPR, 2014.

EDOF problem : Joint optimization of the sensor position and the restoration network within a depth range [20 to 50 m]





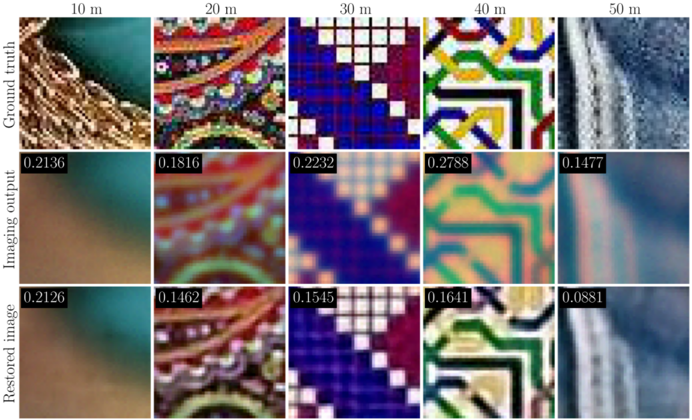
⇒ Validation of the DRT model and the network joint optimization
⇒ The focus is automatically adapted to the depth range of interest

	Without sensor position optimization		With sensor position optimization	
Object position	RMSE	MAE	RMSE	MAE
10m	**0.1032**	**0.0667**	0.1169	0.0774
15m	**0.0503**	**0.0317**	0.0841	0.0530
20m	0.0503	0.0320	**0.0468**	**0.0298**
25m	0.0678	0.0430	**0.0490**	**0.0310**
30m	0.0786	0.0499	**0.0488**	**0.0312**
35m	0.0833	0.0528	**0.0478**	**0.0298**
40m	0.0876	0.0555	**0.0482**	**0.0309**
45m	0.0919	0.0586	**0.0563**	**0.0356**
50m	0.0955	0.0614	**0.0697**	**0.0392**
Average	0.0787	0.0502	**0.0631**	0.0398

Table: RMSE stands for root mean square error and MAE for mean absolute error.

Application

Restoration qualitative results



⇒ Restoration quality is visible on image patch

Contents

Conclusions and perspectives

Conclusion

- Validation the DRT model and neural network joint optimization
- Application in a single parameter optimisation for EDOF

Parallel work

Use of differential ray tracing in co-design of complex optical system and neural network [26]

Perspectives

- Optimization of several optical parameters
- Variation of the PSF off axis
- Application to 3D : see **R. Leroy's** talk !

[26] Sun et al, ACM SIGGRAPH, 2021.

Open questions on Deep Codesign

Open questions

- How do optical and processing parameters interact during the joint optimization ?
- What image quality (i.e. type of aberration) will be acceptable by the network for a given task ?
- How to ensure a physically realistic solution ?
- How important is the optical system starting point of the optimization ?
- Can this approach generate unconventional and original system ?
- How to model the performance ?

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Thank you for your attention

Do you have any question ?

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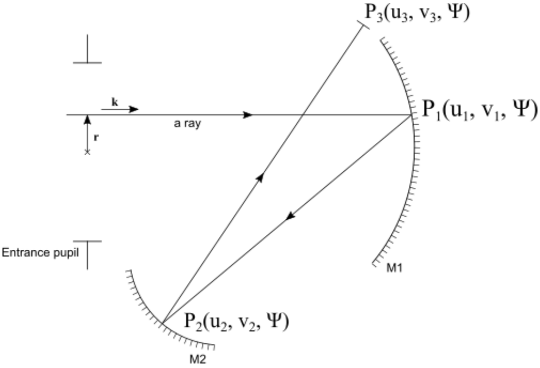
DRT model
Principle

Evaluate the derivative of the ray propagation with respect to "system" parameters



Ψ contains curvatures and positions of the mirrors for instance.

Sum of optical paths :

$$L = \sum_i n_{i,i+1} ||P_i P_{i+1}|| \quad (1)$$

Fermat principle : $\frac{\partial L}{\partial u_i} = 0$ and $\frac{\partial L}{\partial v_i} = 0$

$$n_{i-1,i} \frac{\partial ||P_{i-1} P_i||}{\partial u_i} + n_{i,i+1} \frac{\partial ||P_i P_{i+1}||}{\partial u_i} = f_{u,i} = 0 \quad (2)$$

(idem for ν_i). For any system parameter Ψ_j:

$$\frac{df_{u,i}}{d\Psi_j} = \frac{\partial f_{u,i}}{\partial \Psi_j} + \sum_{k=0}^{N} \frac{\partial f_{u,i}}{\partial u_k} \frac{\partial u_k}{\partial \Psi_j} + \sum_{k=0}^{N} \frac{\partial f_{u,i}}{\partial v_k} \frac{\partial v_k}{\partial \Psi_j} = 0 \quad (3)$$

The derivatives $\frac{\partial f_{u,i}}{\partial u_k}$ and $\frac{\partial f_{u,i}}{\partial \Psi_j}$ can be calculated by automatic differntiation (Julia). Solving this linear system yields : $\frac{\partial u_k}{\partial \Psi_j}$